

Thermal lensing-induced bifocusing of spatial solitons in Kerr-type optical media

Alain M. Dikandé

*Laboratory of Research on Advanced Materials and Nonlinear Sciences (LaRAMaNS),
Department of Physics, Faculty of Science, University of Buea PO Box 63 Buea, Cameroon
(Dated: January 12, 2013)*

Thermo-optical effects cause a bifocusing of incoming beams in optical media, due to the birefringence created by a thermal lens that can resolve the incoming beams into two-component signals of different polarizations. We propose a non-perturbative theoretical description of the process of formation of double-pulse solitons in Kerr optical media with a thermally-induced birefringence, based on solving simultaneously the heat equation and the propagation equation for a beam in a one-dimensional medium with uniform heat flux load. By means of a non-isospectral Inverse Scattering Transform assuming an initial solution with a pulse shape, a one-soliton solution to the wave equation is obtained that represents a double-pulse beam whose characteristic properties depend strongly on the profile of heat spatial distribution.

PACS numbers: 42.65.Tg, 42.70.Gi, 44.10.+i

In dielectric media with thermo-optical effects, the modulations of incoming beams can lead to a wavefront distortion [1–3] reflecting their instability. Generally this instability gives rise to a depolarization of a high-power field [4–9] due to a thermally-induced birefringence which is attributed [10–12] to a change in the refractive index of the medium. It has been established [1] that this change in the refractive index originates from heat deposition in the propagation medium, resulting in a space-dependent temperature gradient (so-called thermal lensing) [1, 4, 13]. For media with linear indices, the thermal lens leads to a drastic change in the irradiance along the beam axis so that the resulting depolarization can strongly degrade the beam quality requiring thermal lensing compensation. However in nonlinear media such as Kerr media, the nonlinearity can be a relevant self-compensation factor [14] stabilizing multi-wave modes generated by the thermal birefringence.

While several materials exhibiting thermo-optical effects are known in the literature [1], a most investigated one is the solid-state laser Nd:YAG [1, 4, 13, 15]. This material is represented as a rod crystal with a cylindrical geometry, where the change in temperature induces thermal distortion of incoming laser beams [9]. For this particular material, several theoretical attempts have been made to formulate the spatial profile of the temperature gradient along the rod exploiting available experimental data. In particular, in refs. [1, 13, 15] it was found that in the cylindrical rod configuration where the heat is generated at a constant rate [9, 13], a quadratic spatial distribution gives a very good description of the experimentally observed birefringence and the resulting beam bifocusing [9, 16].

But thermo-optical processes are actually common to a broad class of materials, not just solid-state lasers. Indeed, photonic crystals and optical fibers (including laser fibers) displaying nonlocal thermal and photothermal properties have been considered in the recent past, from both experimental and theoretical points of view [17, 18]. These materials share in common the fact that heat re-

sulting from the input pump source causes physical variation of the material. Namely, the material expands with the heat load due to a stress gradient formed which produces space-dependent birefringence in the material. Quite remarkably when the thermal gradient is strong enough the thermally-induced birefringence can resolve a polarized high-power input beam into two-component beams [4, 9, 19], moreover upon recombination after traversing the bulk the two beams with two polarization directions are no longer in phase with one another such that the polarization state of the input beam cannot be recovered [3, 19].

The thermally-induced depolarization phenomenon together with the resulting beam bifocusing have been investigated experimentally and theoretically by several authors [4–9]. In photonic crystals with a nonlinear Kerr-type dielectric susceptibility competing with diffraction, nonlocal thermal properties of the propagation medium have very recently been shown to induce twin-mode spatial solitons so-called dipole solitons [20–23]. The robustness of these double-pulse structures provides strong indication that the competition between nonlinearity and thermal lensing might be a stabilizing factor for the double-polarized modes, potentially observable in several optical materials including solid-state and optical fibers with thermally-induced birefringence.

The present work aims at proposing a non-perturbative description of the generation of a double-polarized pulse beam in optical materials with thermally-induced linear birefringence, paying particular attention to nonlinear optical media with a quadratic spatial profile of temperature distribution. Our objective is to provide a consistent theoretical framework for understanding better the effects of thermal lensing on beam propagation in nonlinear optical materials, of which the YAG fiber laser whose temperature profile was experimentally established [1, 13, 15]. We will first demonstrate, by solving the heat equation, that the topography of temperature distribution is quadratic in the case of a one-dimensional (1D) anisotropic material uniformly loaded at a constant

heat flux rate. Next, using the relationship between the temperature gradient and the induced inhomogeneous optical index, we shall derive an effective refractive index for the material by combining the thermally-induced optical index and a Kerr-type refractive index reflecting the intrinsic nonlinearity of optical properties of the 1D anisotropic material. With the help of this effective refractive index we shall formulate the propagation equation for beams in the thermal nonlinear medium. Note that this equation is a Nonlinear Schrödinger equation (NLSE) with a repulsive external quadratic potential, that has already been derived in ref. [24] but solved perturbatively. Here instead, a non-perturbative treatment will be proposed based on a non-isospectral Inverse Scattering Transform (IST) method [25, 26] with emphasis on IST's initial solution being single-pulse shaped. Thermal lensing can be described in simple words as a thermo-optical process associated with a weak absorption of an input beam that induces a nonzero temperature gradient across the material, leading to a spatial variation of its refractive index [1, 13, 21]. Recent experimental as well as theoretical developments on this process suggest that the underlying mechanism involves a local refractive index change $\Delta n(x)$ which increases linearly [15, 21] with the temperature change $\Delta T(x)$ i.e. $\Delta n = \beta \Delta T$, where $\beta = dn/dT$ refers to the thermo-optic coefficient [21, 27]. Thus when the optical beam of a uniform thermal load gets slightly absorbed and heats the material, this produces heat that is conveyed by the electromagnetic wave. If ρ denotes the uniform heat flux density and κ the heat conductivity coefficient, the heat diffusion in the material along a preferred direction (for 1D anisotropic materials of current interest), driven by the uniform heat flux load, is determined by the heat equation:

$$\kappa \nabla^2 T(x) = -\rho. \quad (1)$$

Since eq. (1) is key to the current analysis we consider its most general solution given by:

$$T(x) = -\frac{1}{2}a_2 x^2 + a_1 x + T_0, \quad (2)$$

where $a_2 = \rho/\kappa$, a_1 and T_0 being two arbitrary real constants. Formula (2) is consistent with the quadratic law of temperature variation found for most laser fibers in the presence of a uniform head load. More specifically in YAG fiber lasers this law is common [1, 13, 15] and is consistent with the optical bifocality associated with a thermally-induced birefringence, that promotes double-polarized laser beams from an input laser field.

For the sake of simplicity we require the temperature gradient to be zero and temperature to take a bare value T_0 at $x = 0$ (i.e. the ambient temperature). The change of temperature in the material along the x axis, hereafter assumed to be the axis of beam propagation, then reads:

$$\begin{aligned} \Delta T(x) &= T_0 - T(x) \\ &= \frac{1}{2}a_2 x^2. \end{aligned} \quad (3)$$

With the last formula we derive the following expression for the local refractive index change:

$$\Delta n(x) = \frac{1}{2}\alpha x^2, \quad \alpha = \rho\beta/\kappa. \quad (4)$$

Now if the intrinsic optical properties of the material are dominated by Kerr-type phenomena, the homogeneous part of the refractive index can be expressed as $(k^2 c/2\pi)n_2 I$ where I is the beam intensity. With the help of (4) we can readily define an effective refractive index $n(x)$ for the thermal nonlinear material viz:

$$n[x, I] = \Delta n(x) + (k^2 c/2\pi)n_2 I. \quad (5)$$

Assuming that the wave motion is fast along the axis of anisotropy (i.e. x) but very slow along z [21], the paraxial approximation on the 2D wave equation for an electromagnetic field $q(x, z)$ leads to:

$$\frac{\partial^2 q}{\partial x^2} + 2ik \frac{\partial q}{\partial z} + n[x, I] q = 0. \quad (6)$$

As already stressed eq. (6) is actually not new, indeed the same equation was obtained [24] for the same problem but solved following the collective-coordinate method. To this last point, eq. (6) is an inhomogeneous NLSE and so can in principle be solved using the collective-coordinate method. However this is a perturbative method and consequently requires that the thermal lensing is sufficiently weak, so that the Kerr nonlinearity remains the main governing factor in the modulation and stability of signals in the thermal nonlinear medium. So to say any input beam sent in the medium must be modulated into a signal of permanent single-pulse shape, with eventually an acceleration or slow down of the pulse due to the thermal lensing. In fact this consideration is very far from any acceptable consistency with the physics of the process under study, which the double-polarization of the incident beam is a most salient aspect.

Being interested in a solution to eq. (6) which is more consistent with experiments we try for a non-perturbative approach. Remark to start that this equation can be rewritten in the following form:

$$\frac{\partial^2 q}{\partial x^2} + 2ik \frac{\partial q}{\partial z} + [(k^2 c/2\pi)n_2 |q|^2 - V(x)] q = 0, \quad (7)$$

corresponding to a NLSE with an external potential

$$V(x) = -\alpha x^2/2 \quad (8)$$

which is quadratic in x with a maximum at $x = 0$. The physics behind this quadratic potential is contained both in its profile and the parameter α defined in (4), which determines the strength of the thermal birefringence on the beam shape. One remarkable side of this physics emerges from the assumption of the heat flux density ρ and the heat conductivity κ as being fixed, such that α appears to

be increasing with an increase of the thermo-optic coefficient β . Thus when β is increased the curvature of the quadratic profile of heat distribution in the material is more and more pronounced so that the effect of thermal lensing on beam modulation is more and more strong. In fact the external potential is expected to be much effective on the beam position and according to the profile of this potential given by (8), the centre of mass of the input beam should experience a trapping force from the manifestly expulsive quadratic potential.

When α is large enough such that the potential field erected by the thermal lensing process on the beam path is strongly localized, its contribution must be fully taken into consideration. In this last respect we follow the non-isospectral IST technique proper [25, 26] to equations of this specific kind, considering an initial signal $q(x, z = 0)$ of a permanent single-pulse shape along x at $z = 0$. For eq. (6) this technique leads to the following one-soliton solution:

$$q(x, z) = Q_0(x, z) \operatorname{sech}[\Psi(x, z)] e^{i\Phi(x, z)}, \quad (9)$$

with

$$Q_0(x, z) = \sqrt{\frac{2\pi}{cn_2}} \frac{x f(z)}{k}, \quad (10)$$

$$\Psi(x, z) = \ln(2|f(z)|) - 2V(x)f(z) + \Psi_0, \quad (11)$$

$$\Phi(x, z) = 2V(x)g(z) + \Phi_0, \quad (12)$$

$$f(z) = \frac{\operatorname{sech}^2(z/\zeta)}{1 + 8(\rho\beta/\kappa)^2 \tanh^2(z/\zeta)}, \quad (13)$$

$$g(z) = \frac{\kappa^2 + 8(\rho\beta)^2}{2\kappa\sqrt{2\rho\beta\kappa}} \frac{\tanh(z/\zeta)}{1 + 8(\rho\beta/\kappa)^2 \tanh^2(z/\zeta)}, \quad (14)$$

$$\zeta = (2/\alpha)^{1/2}k. \quad (15)$$

If the "sech" function in (9) reminds a pulse signal, the complicated dependence of its prefactor $Q_0(x, z)$ in x and z clearly suggests an actually complex pulse structure for the beam on the xz plane. To gain insight about what this dependence implies for the signal profile, in fig. 1 we plotted the beam amplitude $|q(x, z)|$ as a function of x at two different positions in the direction z transverse to the beam propagation, and for four distinct values of α . More explicitly the four left graphs in fig. 1 represent $|q(x, z)|$ versus x at $z = 0.5$ for $\alpha = 0.1, 0.05, 0.01$ and 0.005 , while the left graphs represent $|q(x, z)|$ versus x for same values of α but at $z = 10$. As one sees, the signal intensity is a two-component pulse which intensities are strongly dependent on the strength of the thermal lens potential. It is quite noticeable on exploring the six figures that when α increases, the intensities of the two-component pulse increase while their width at half tails diminish. The last behaviour is consistent with the

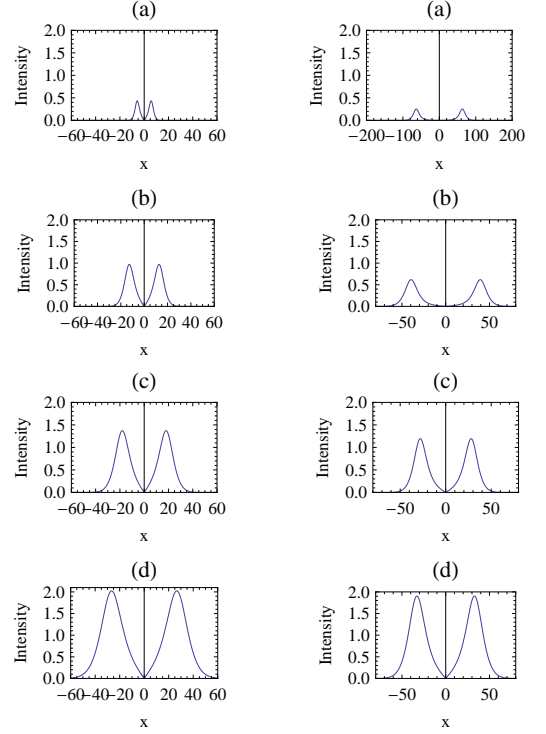


FIG. 1: x -axis profiles of $|q(x, z)|$ for $z = 0.5$ (left graphs) and $z = 10$ (right graphs). Values of α are (a) 0.1, (b) 0.05, (c) 0.01, (d) 0.005

α dependence of parameter ζ defined in (15), which indeed represents the average spatial extension of the pulse along the x axis.

Another relevant feature emerging from graphs of fig. 1 is the fact that when α is decreased for a fixed value of z , the two constituents pulses in the double-polarized beam preserve their shapes but their peak positions (i.e. centres) are gradually shifted. The last feature is more transparent in the left graphs corresponding to a relatively large value of z . In fact, the last behaviour can be interpreted in terms of a ring profile for the signal intensity in the xz plane as reflected by the contour plots of fig. 2, where shadows of the double-pulse soliton in the xz plane are represented for different values of α .

The figure clearly indicates an increase of the separation between pulses in the double-pulse signal, implying that the radius and curvature of the ring signal in the xz plane are fixed by the magnitude of α . The double-pulse structure obtained as well as its ring profile emerging in 2D are reminiscent of dipole and ring-vortex solitons, predicted recently in some nonlocal nonlinear media [21, 23, 28]. However in these previous studies the physical origins of nonlocalities were generally not well specified, whereas in our context the quadratic form of the optical inhomogeneity has an experimental foundation [1, 13, 15]. Also, a previous attempt [24] to model the same problem led to eq.(6). In this previous work the equation was treated pertur-

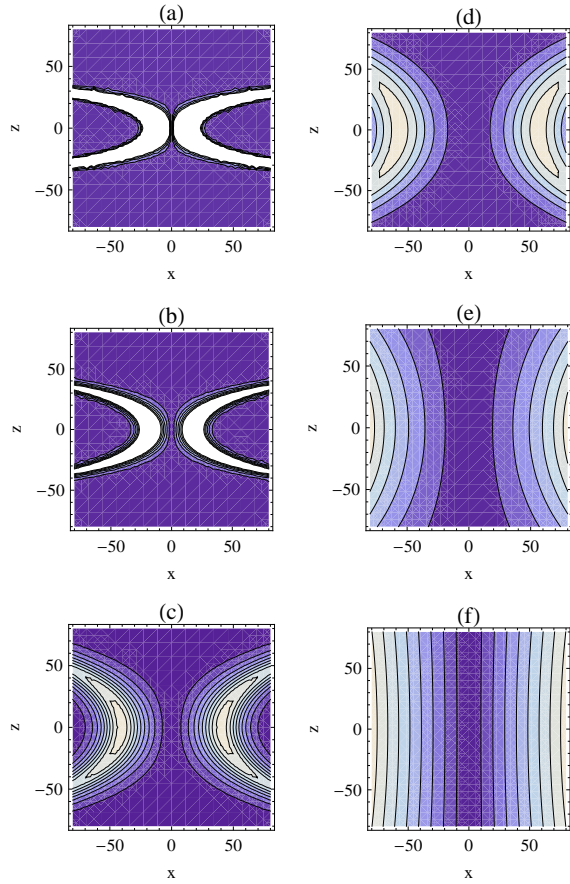


FIG. 2: Contour plots of $|q(x, z)|$ for different α listed as: (a) 0.01, (b) 0.005, (c) 0.001, (d) 0.0005, (e) 0.0001, (f) 0.00001.

batively and so results could not reflect the remarkable aspects underlying the physics of thermal lensing in Kerr media. As end remark, the universality of the IST one-soliton solution (9) can be checked by applying any other exact spectral method to the generating equation, such as the Darboux method [29] combined with a Lax-pair formalism with non-conserved spectral parameters [30, 31].

Part of this work was done at the Abdus Salam International Centre for Theoretical Physics (ICTP) Trieste, Italy. I thank Markus Muller, Matteo Marsili, M. Kiselev and V. Kravtsov for their kind hospitalities.

-
- [1] W. Koechner, *Solid State Laser Engineering* (Springer-Berlin, 6th edition, 2006).
 - [2] J. D. Foster and L. M. Osterink, J. Appl. Phys. **41**, 3656 (1970).
 - [3] C. A. Klein, Opt. Eng. **29**, 343 (1990).
 - [4] H. J. Eichler, A. Hasse, R. Menzel and A. Siemoneit, J. Phys. D **26**, 1884 (1993).
 - [5] M. Ostermeyer, G. Klemz and R. Menzel, Proc. SPIE **4629**, 67 (2002).
 - [6] I. Moshe and S. Jackel, J. Opt. Soc. Am. B **22**, 1228 (2005).
 - [7] G. Machavariani, Y. Lumer, I. Moshe, A. Meir, S. Jackel and N. Davidson, Appl. Opt. **46**, 3304 (2007).
 - [8] Y. Wang, K. Inoue, H. Kan and S. Wada, J. Phys. D **42**, 235108 (2009).
 - [9] Y. Wang, H. Kan, T. Ogawa and S. Wada, J. Opt. **13**, 015703 (2011).
 - [10] E. A. Khazanov, O. V. Kulagin, S. Yoshida, D. B. Tanner and D. H. Reitze, IEEE Quantum Electron. **35**, 1116 (1999).
 - [11] F. Genereux, S. W. Leonard and H. M. Driel, Phys. Rev. B **63**, 161101(R) (2001).
 - [12] I. Shoji, Y. Sato, S. Kurimura, V. Lupei, T. Taira, A. Ikesue and K. Yoshida, Opt. Lett. **27**, 234 (2002).
 - [13] W. Koechner, Appl. Opt. **9**, 2548(1970).
 - [14] A. S. Koujelev and A. E. Dudelzak, Opt. Eng. **47**, 085003(2008).
 - [15] H. J. Eichler, A. Haase, R. Menzel and A. Siemoneit, J. Phys. D **26**, 1884 (1993).
 - [16] M. Frede, R. Wilhelm, M. Brendel, C. Fallnich, F. Seifert, B. Willke and K. Danzmann, Optics Express **12**, 3581 (2004).
 - [17] S. Vasudevan, G. C. K. Chen and B. S. Ahluwalia, Opt. Lett. **33**, 2779 (2008).
 - [18] M. Andika, G. C. K. Chen and S. Vasudevan, J. Opt. Soc. Am. B **27**, 796 (2010).
 - [19] S. Wielandy and A. L. Gaeta, Phys. Rev. Lett. **81**, 3359(1998).
 - [20] A. I. Yakimenko, Y. A. Zaliznyak and Y. Kivshar, Phys. Rev. E **71**, 065603 (2005).
 - [21] C. Rotschild, O. Cohen, O. Manela, M. Segev and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
 - [22] S. Skupin, M. Saffman and W. Królkowski, Phys. Rev. Lett. **98**, 263902 (2007).
 - [23] F. Ye, Y. V. Kartashov, Bambi Hu and L. Torner, Opt. Lett. **34**, 2658 (2009).
 - [24] A. Gharaati, P. Elahi and S. Cari, Act. Phys. Pol. A **112**, 891 (2007).

- [25] R. Balakrishnan, Phys. Rev. B **32** 1144 (1985).
- [26] A. M. Dikandé, J. Math. Phys. **49**, 073520 (2008).
- [27] R. Sowade, I. Breunig, C. Tulea and K. Buse, Appl. Phys. B **99**, 63 (2010).
- [28] F. Ye, B. A. Malomed, Y. He and Bambi Hu, Phys. Rev. A **81**, 043816 (2010).
- [29] Z. Xu, L. Li, Z. Li, G. Zhou and K. Nakkeeran, Phys. Rev. E **68**, 046605(2003).
- [30] R. Rhada, V. R. Kumar and K. Porsezian, J. Phys. A **41**, 315209 (2008).
- [31] U. Al Khawaya, J. Phys. A **39**, 9679 (2006) & A **42**, 265206 (2009).